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Decentralized Adaptive Robust Control and Its Applications to an Uncertain Flexible Beam

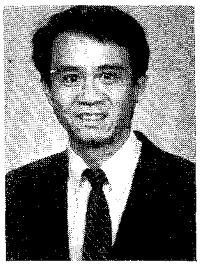
Y. H. Chen
Georgia Institute of Technology, Atlanta, Georgia 30332
and
E. D. Piontek
Naval Air Test Center, Patuxent River, Maryland 20670

We consider a class of nonlinear uncertain interconnected systems with time-varying uncertainty. The uncertainty may arise within each system as well as in the interconnections. The uncertainty is assumed to be bounded but the bound is unknown. No a priori statistical information is imposed. Decentralized adaptive robust control is proposed for each system. The control has two parts. First, an adaptive scheme for the estimation of the bound is constructed. Second, a robust control, which is based on the adaptive parameter, is adopted for each system. The control is then applied to an uncertain flexible beam.

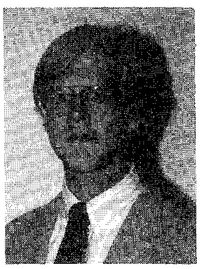
I. Introduction

A CLASS of decentralized adaptive robust control has been first designed for uncertain large-scale dynamic systems consisting of interconnected subsystems. The uncertainty under consideration is due to internal parameter variation, external excitation, unknown nonlinearity, and unknown interconnection. The uncertainty is nonlinear and time varying. The distinct feature of the control is that it does not require any statistical information of the uncertainty. Instead, only information related to the bound of the uncertainty is utilized. However, this does not imply that the possible bound of the uncertainty needs to be determined a priori. The control design is divided into two parts. First, a decentralized adaptive scheme for on-line estimation of the bound is constructed. Second, robust control that is based on this estimation is used. The resulting overall system performance is described deterministically. This implies that what is prescribed is guaranteed. An early work along this line is Ref. 1. The specific advantages of the current design are twofold. First, comparing with Ref. 1, the control effort is significantly reduced. This is because the bound is to be determined a priori in Ref. 1 and, therefore, overconservative estimation becomes inevitable. Besides, this also decreases the required information about the uncertainty for the control design. In practical applications, estimating the bound of the uncertainty has to do with studying the physical causes of the uncertainty. Second, the control is now applicable to a class of uncertain flexible structures. The previous work[1] is not applicable due to the need of excessively large control effort.

Subduing large flexible system response by active control has been an important control problem in the recent past.[2] There has been a significant amount of research in this area.[3-15] We propose to use the decentralized adaptive robust control for this problem. The emphasis is on appropriate active control design that can compensate the modeling uncertainty. Specifically, this refers to the flexible structure that



Ye-Hwa Chen was born in Taiwan, R.O.C. He received his B.S. degree in chemical engineering from National Taiwan University in 1979 and the MS. and Ph.D. degrees in mechanical engineering from the University of California, Berkeley, in 1983 and 1985. From 1986 to 1988, he served as a faculty member in the Department of Mechanical and Aerospace Engineering at Syracuse University. He then became a faculty member of the School of Mechanical Engineering and the Laboratory of Intelligent Control Systems of the Georgia Institute of Technology, Atlanta. His research has been on advanced control methods for mechanical systems, adaptive robust control of uncertain systems, large-scale systems, discrete events, and intelligent control. Dr. Chen is a consultant to industry in matters of automation. He is the author of more than 60 refereed papers. Dr. Chen has established and served as the chairman for the Complex Systems Technical Panel of the American Society of Mechanical Engineers since 1987.



E. Piontek is currently employed by the Department of Defense at the Naval Air Test Center as a mechanical engineer. His work consists of designing experimental and test related devices for naval aircraft. He is also currently studying for an M.S. degree in Engineering Management at the Florida Institute of Technology (adjunct). He holds M.S. and B.S. degrees in Mechanical Engineering from Syracuse University, and a B.A. in Physics from the State University of New York at Plattsburgh. His career interests are in advancing engineering technologies in the areas of dynamics and materials. He is a Member of the AIAA.

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contains unknown deviation in its mass and stiffness distributions and possesses terms that represent disturbance action. The known structure could degenerate into an uncertain structure due to, for example, changes in the environment or inaccuracies in the contruction that will then alter the system's mass and stiffness distributions. In addition, unpredicted disturbance forces could act on the structure.

To monitor the performance of a flexible structure with this decentralized adaptive robust control, computer simulations were performed. A postulated tapered cantilever beam was used to design a control for an actual tapered cantilever beam with uncertain mass and stiffness variations, as well as disturbance force action. The actual structure is truncated to include the first seven modes, and an adaptive robust modal control is designed for each mode.

II. Decentralized Adaptive Robust Control Design

We first propose a class of decentralized adaptive robust controls for interconnected uncertain systems. The purpose of this section is first to consider a general situation of uncertain systems.

Consider the following class of interconnected uncertain systems:

$$S_i : \dot{x}_i(t) = A_i x_i(t) + \Delta f_i\big(x_i(t),t\big) + \big\{B_i + \Delta B_i\big(x_i(t),t\big)\big\} u_i(t) + \sum_{j=1,j\neq i}^{N} g_{ij}\big(x_j(t),t\big) \tag{1}$$

$$x_i(t_0) = x_{i0}$$

where $i = 1,2,\ldots,N$. Here, $t \in \boldsymbol{R}$, $x_i(t) \in \boldsymbol{R}^{n_i}$ is the state, and $u_i(t) \in \boldsymbol{R}^{m_i}$ is the control. Matrices (vectors) A_i, $\Delta f_i(x_i,t)$, B_i, $\Delta B_i(x_i,t)$, and $g_{ij}(x_j,t)$ are of appropriate dimensions. The unknown nonlinear functions $\Delta f_i(\cdot) : \boldsymbol{R}^{n_i} \times \boldsymbol{R} \to \boldsymbol{R}^{n_i}$, $\Delta B_i(\cdot) : \boldsymbol{R}^{n_i} \times \boldsymbol{R} \to \boldsymbol{R}^{n_i \times m_i}$, $g_{ij}(\cdot) : \boldsymbol{R}^{n_j} \times \boldsymbol{R} \to \boldsymbol{R}^{n_i}$ are continuous. These represent parameter variation, unknown nonlinearity, and external excitation. Note that the input of S_j, namely, u_j, does not explicitly enter S_i. Rather, it dictates the state x_j, which then affects x_i. This stands for the situation that the system can be governed in a decentralized manner.[16]

The problem is to design the control for each system S_i such that the overall large-scale system possesses certain desirable properties. However, the control should not be based on any stochastic property of the uncertainty Δf_i, ΔB_i, and g_{ij}.

The following assumptions for S_i are imposed.

Assumption 1: The pair (A_i, B_i) can be stabilized.

Assumption 2: There exist an (unknown) constant $\theta_i > -1$ and (unknown) functions $h_i(\cdot)$ and $E_i(\cdot)$ such that for all $(x_i,t) \in \boldsymbol{R}^{n_i} \times \boldsymbol{R}$,

$$\Delta f_i(x_i,t) = B_i h_i(x_i,t) \tag{2}$$

$$\Delta B_i(x_i,t) = B_i E_i(x_i,t) \tag{3}$$

$$\min_{x,t} \tfrac{1}{2}\lambda_m\big[E_i(x_i,t) + E_i^T(x_i,t)\big] \geq \theta_i \tag{4}$$

Here, $\lambda_m(\cdot)$ is the minimum eigenvalue of the designated matrix. Similarly, we shall use $\lambda_M(\cdot)$ to denote the maximum eigenvalue of the designated matrix. Implicitly assumed is that all eigenvalues are real.

Remark 1: Assumption 1 implies that there exists a matrix K_i such that $\bar{A}_i$ is Hurwitz where $\bar{A}_i = A_i + B_i K_i$. Assumption 2 imposes structural restriction on the uncertain portion. This is essential since no other assumption on the realization of the uncertain portion is imposed. Equations (2) and (3) are usually referred to as the matching conditions. The vector $\Delta f_i(x_i,t)$ and the matrix $\Delta B_i(x_i,t)$ should lie within the range space of the nominal input matrix B_i. Inequality (4) implies that the uncertain input matrix ΔB_i should not be too large in the opposite direction of the one that is specified by the nominal input matrix B_i. This assures that any proposed control action can act in the desired direction. However, the control magnitude in this direction is unknown. Studies of the relaxations of this assumption can be found in, e.g., Ref. 17. A measure of mismatch is proposed to indicate the allowable deviation from this assumption. The control proposed in this paper is applicable to the mismatched system. However, the proof is restricted to this assumption for brevity.

Assumption 3: 1) There exist an unknown constant vector $\beta_i \in (0,\infty)^{s_i}$ and a known function $\rho_i(\cdot) : \boldsymbol{R}^{n_i} \times (0,\infty)^{s_i} \times \boldsymbol{R} \to \boldsymbol{R}_+$ such that for all $(x_i,t) \in \boldsymbol{R}^{n_i} \times \boldsymbol{R}$,

$$(1+\theta_i)^{-1} \| h_i(x_i,t) + E_i(x_i,t) K_i x_i \| \leq \rho_i(x_i,\beta_i,t) \tag{5}$$

2) The function $\rho_i(x_i,\cdot,t) : (0,\infty)^{s_i} \to \boldsymbol{R}_+$ is C^1 and concave; hence, for any $\beta_{i1},\beta_{i2} \in (0,\infty)^{s_i}$,

$$\rho_i(x_i,\beta_{i1},t) - \rho_i(x_i,\beta_{i2},t) \leq \frac{\partial \rho_i^T}{\partial \beta_i}(x_i,\beta_{i2},t)(\beta_{i1} - \beta_{i2}) \tag{6}$$

Throughout, we shall use the Euclidean vector norm. The matrix norm is then the corresponding induced one. Hence, for a given matrix Υ, $\|\Upsilon\|^2 = \lambda_M(\Upsilon^T\Upsilon)$.

Remark 2: The unknown parameter β_i in ρ_i is related to the bound of uncertainty. This is, however, interpreted in a less rigorous way. Notice that the dimension of β_i (i.e., s_i) can be different from that of the uncertain portion Δf_i, etc. Moreover, the verification of this assumption does not require the knowledge of the true value of β_i. Since the function $\rho_i(\cdot)$, which satisfies Eqs. (5) and (6), is nonunique, it is then the designer's discretion for a simplified version of $\rho_i(\cdot)$. For example, if $\|h_i(x_i,t)\| \leq \beta_{1i}\|x_i\| + \beta_{2i}$ and $\|E_i(x_i,t)\| \leq \beta_{3i}$, then one may choose

$$\rho_i(x_i,\beta_i,t) = (1+\theta_i)^{-1}\big(\beta_{1i} + \beta_{3i}\|K_i\|\big)\|x_i\| + (1+\theta_i)^{-1}\beta_{2i}$$

$$\beta_i = [\beta_{1i} \;\; \beta_{2i} \;\; \beta_{3i}]^T$$

This results in $s_i = 3$. A simpler choice can be

$$\rho_i(\xi,\beta_i,t) = \beta_{1i}\|x_i\| + \beta_{2i}$$

where $\beta_i = [\beta_{1i} \;\; \beta_{2i}]^T$. This results in $s_i = 2$. This choice is also advantageous for the simplification of the adaptive scheme design [shown in Eq. (8)].

Assumption 4: There exist non-negative constants a_{ij} and b_{ij} such that for all $(x_j,t) \in \boldsymbol{R}^{n_j} \times \boldsymbol{R}$,

$$(1+\theta_i)^{-1}\|g_{ij}(x_j,t)\| \leq a_{ij}\|x_j\| + b_{ij} \tag{7}$$

This assumption implies that the (unknown) interconnections are cone bounded. Thus, the bound is linear. However, it is not necessary for $g_{ij}(x_j,t)$ to be linear.

Since the constant vector β_i is unknown, it is then desirable to construct an adaptive scheme. Consider the decentralized adaptive scheme

$$\dot{\hat{\beta}}_i(t) = L_{i1}\big\|\alpha_i\big(x_i(t)\big)\big\|\frac{\partial}{\partial \beta_i}\rho_i^T\big(x_i(t),\hat{\beta}_i(t),t\big) - L_{i2}\hat{\beta}_i(t) \tag{8}$$

where $\alpha_i(x_i) = B_i^T P_i x_i$, P_i is the solution of the Lyapunov equation $\bar{A}_i^T P_i + P_i \bar{A}_i + Q_i = 0$, $Q_i > 0$, and L_{i1} and L_{i2} are $s_i \times s_i$ positive definite matrices.

It is worth noting that the adaptive scheme (8) is of a leakage type. This is shown by the second term, which indicates the amount of leakage. The first term is positive and reflects the need of higher bound if the state x_i is far away from the origin. The matrix L_{i1} is the learning rate. Further discussion is deferred to Remark 4.

For given $\epsilon_i > 0$, $i = 1,2,\ldots,N$, the decentralized adaptive robust control scheme is now proposed:

$$u_i(t) = K_i x_i(t) + p_i\big(x_i(t),\hat{\beta}_i(t),t\big) \tag{9}$$

where

$$p_i(x_i,\hat\beta_i,t) = -\frac{\mu_i(x_i,\hat\beta_i,t)}{\|\mu_i(x_i,\hat\beta_i,t)\|}\,\rho_i(x_i,\hat\beta_i,t) \quad \text{if } \|\mu_i(x_i,\hat\beta_i,t)\| > \epsilon_i \tag{10a}$$

$$p_i(x_i,\hat\beta_i,t) = -\frac{\mu_i(x_i,\hat\beta_i,t)}{\epsilon_i}\,\rho_i(x_i,\hat\beta_i,t) \quad \text{if } \|\mu_i(x_i,\hat\beta_i,t)\| \le \epsilon_i \tag{10b}$$

$$\mu_i(x_i,\hat\beta_i,t) = \alpha_i(x_i)\rho_i(x_i,\hat\beta_i,t) \tag{11}$$

We summarize the design steps for the decentralized adaptive robust controls as follows.

Step 1: Based on the uncertain systems (1), select the gain matrix K_i such that the matrix $\bar A_i = A_i + B_iK_i$ is Hurwitz.

Step 2: Solve the Lyapunov equation for each i and then obtain α_i.

Step 3: Obtain the function $\rho_i(x_i,\beta_i,t)$ that meets Eqs. (5) and (6).

Step 4: Choose $L_{i1}>0$ and $L_{i2}>0$. Construct the adaptive scheme (8).

Step 5: Choose $\epsilon>0$. Construct the robust control (9) with $\mu_i = \alpha_i\rho_i$.

Next we discuss the resulting (controlled) system performance. Let

$$x = \left[x_1^T, x_2^T, \ldots, x_N^T\right]^T \tag{12}$$

$$\hat\beta = \left[\hat\beta_1^T, \hat\beta_2^T, \ldots, \hat\beta_N^T\right]^T \tag{13}$$

$$\beta = \left[\beta_1^T, \beta_2^T, \ldots, \beta_N^T\right]^T \tag{14}$$

$$x(t_0) = x_0 = \left[x_{10}^T, x_{20}^T, \ldots, x_{N0}^T\right]^T \tag{15}$$

$$\hat\beta(t_0) = \hat\beta_0 = \left[\hat\beta_{10}^T, \hat\beta_{20}^T, \ldots, \hat\beta_{N0}^T\right]^T \tag{16}$$

and, hence, $x \in \boldsymbol{R}^n$, $n = \Sigma_{i=1}^N n_i$, $\hat\beta \in \boldsymbol{R}^s$, $\beta \in \boldsymbol{R}^s$, and $s = \Sigma_{i=1}^N s_i$. Consider also the following test matrix

$$T = \left[t_{ij}\right]_{N\times N} \tag{17}$$

where

$$t_{ij} = \begin{Bmatrix} \lambda_m(Q_i), & i=j \\ -2(1+\theta_i)\lambda_M(P_i)a_{ij}, & i\ne j \end{Bmatrix} \tag{18}$$

Theorem 1: Consider the interconnected systems S_i, $i=1, 2,\ldots,N$, shown in Eq. (1). Suppose that Assumptions 1–4 are met. Suppose that the decentralized adaptive scheme (8) and the robust control (9) are adopted for each system S_i. Then the following properties hold, provided the successive principal minors of the test matrix T are all positive.

1) *Existence of solutions.* Given $(x_0,\hat\beta_0,t_0) \in \boldsymbol{R}^n \times (0,\infty)^s \times \boldsymbol{R}$, the closed-loop system

$$\begin{aligned}\dot x_i(t) &= A_ix_i(t) + \Delta f_i\big(x_i(t),t\big) + \big\{B_i + \Delta B_i\big(x_i(t),t\big)\big\} \\ &\quad\times \big\{K_ix_i(t) + p_i\big(x_i(t),\hat\beta_i(t),t\big)\big\} + \sum_{j=1}^N g_{ij}\big(x_j(t),t\big)\end{aligned} \tag{19}$$

$$\dot{\hat\beta}_i(t) = L_{i1}\big\|\alpha_i\big[x_i(t)\big]\big\|\frac{\partial}{\partial\hat\beta_i}\rho_i^T\big(x_i(t),\hat\beta_i(t),t\big) - L_{i2}\hat\beta_i(t) \tag{20}$$

where $i=1,2,\ldots,N$ possesses a solution $[x(\cdot),\hat\beta(\cdot)] : [t_0,t_1) \to \boldsymbol{R}^n\times(0,\infty)^s$, $[x(t_0),\hat\beta(t_0)] = (x_0,\hat\beta_0)$.

2) *Uniform boundedness.* For each $r_1,r_2>0$, there exist $d_1(r_1,r_2)\ge 0$, $d_2(r_1,r_2)\ge 0$ such that for every solution $[x(\cdot),\hat\beta(\cdot)] : [t_0,t_1) \to \boldsymbol{R}^n\times(0,\infty)^s$,

$$\|x_0\| \le r_1, \quad \|\hat\beta_0 - \beta\| \le r_2 \Rightarrow \|x(t)\| \le d_1(r_1,r_2), \quad \|\hat\beta(t)-\beta\| \le d_2(r_1,r_2) \tag{21}$$

for all $t \in [t_0,t_1)$.

3) *Extension of solution.* Every solution $[x(\cdot),\hat\beta(\cdot)]$: $[t_0,t_1) \to \boldsymbol{R}^n\times(0,\infty)^s$ of Eqs. (19) and (20) can be extended over $[t_0,\infty)$.

4) *Uniform ultimate boundedness.* There exists a constant $\underline{d}>0$ such that the following is true: Given any $\bar d > \underline{d}$ and any $r\in(0,\infty)$, there is a $T(\bar d,r) \in [0,\infty)$ such that for every $[x(\cdot),\hat\beta(\cdot)] : [t_0,\infty) \to \boldsymbol{R}^n\times(0,\infty)^s$,

$$\|\xi_0\| \le r \Rightarrow \|\xi(t)\| \le \bar d \quad \text{for all} \quad t \ge t_0 + T(\bar d,r) \tag{22}$$

where $\xi = [x^T, \hat\beta^T - \beta^T]^T$, $\xi_0 = [x_0^T, \hat\beta_0^T - \beta^T]^T$.

5) *Uniform stability.* Given any $\bar d > \underline{d}$, there exists a $\delta(\bar d) > 0$ such that for every $[x(\cdot),\hat\beta(\cdot)] : [t_0,\infty) \to \boldsymbol{R}^n\times(0,\infty)^s$,

$$\|\xi_0\| \le \delta(\bar d) \Rightarrow \|\xi(t)\| \le \bar d \qquad \text{for all} \quad t \ge t_0 \tag{23}$$

Proof: Property 1 is readily shown by the assumed properties of the right-hand side of Eqs. (19) and (20).[18] Let the Lyapunov function candidate $V(x,\hat\beta-\beta)$ be given by

$$V(x,\hat\beta-\beta) = \sum_{i=1}^N \gamma_iV_{i1}(x_i) + \sum_{i=1}^N \gamma_iV_{i2}(\hat\beta_i-\beta_i) \tag{24}$$

$$V_{i1}(x_i) = x_i^TP_ix_i, \quad V_{i2}(\hat\beta_i-\beta_i) = (1+\theta_i)(\hat\beta_i-\beta_i)L_{i1}^{-1}(\hat\beta_i-\beta_i) \tag{25}$$

where γ_is are chosen in such a way that for $\Omega \triangleq \mathrm{diag}\{\gamma_1,\gamma_2,\ldots,\gamma_N\}$, $2S \triangleq \Omega T + T^T\Omega$ is positive definite. This is always possible provided the successive principal minors of T are all positive.[19,20]

Upon using Assumptions 1 and 2, Eq. (19) can be rewritten as

$$\dot x_i = \bar A_ix_i + B_i[I_i+E_i]p_i + B_ie_i + \sum_j g_{ij} \tag{26}$$

where

$$e_i \triangleq h_i + E_iK_ix_i \tag{27}$$

Throughout the proof, arguments are sometimes omitted for brevity when no confusions are likely to arise. Hence, almost everywhere on $[t_0,t_1)$,

$$\begin{aligned}\dot V_{i1}\big[x_i(t)\big] &= 2x_i^TP_i\dot x_i \\ &= x_i^T(\bar A_i^TP_i + P_i\bar A_i)x_i + 2x_i^TP_iB_i(I_i+E_i)p_i \\ &\quad + 2x_i^TP_iB_ie_i + 2x_i^TP_i\sum_j g_{ij}\end{aligned} \tag{28}$$

By using the Lyapunov equation and the Rayleigh's principle,[21]

$$x_i^T(\bar A_i^TP_i + P_i\bar A_i)x_i = -x_i^TQ_ix_i \le -\lambda_m(Q_i)\|x_i\|^2 \tag{29}$$

In the control portion, if $\|\mu_i\| > \epsilon_i$

$$\begin{aligned}x_i^TP_iB_i(I_i+E_i)p_i &= x_i^TP_iB_i(I_i+E_i)\left(-\frac{\mu_i}{\|\mu_i\|}\rho_i\right) \\ &\le (-\mu_i^T\mu_i)\|\mu_i\|^{-1} \\ &\quad - \tfrac{1}{2}\mu_i^T(E_i+E_i^T)\mu_i\|\mu_i\|^{-1} \\ &\le -\|\mu_i\|^2\|\mu_i\|^{-1} - \theta_i\|\mu_i\| \\ &= -(1+\theta_i)\|\mu_i\|\end{aligned} \tag{30}$$

If $\|\mu_i\| \le \epsilon_i$,

$$x_i^T P_i B_i (I_i + E_i) p_i = x_i^T P_i B_i (I_i + E_i)\left(-\frac{\mu_i}{\epsilon_i}\rho_i\right)$$
$$\le (-\mu_i^T \mu_i)\epsilon_i^{-1} - \tfrac{1}{2}\mu_i^T (E_i + E_i^T)\mu_i \epsilon_i^{-1}$$
$$\le -\|\mu_i\|^2 \epsilon_i^{-1} - \theta_i \|\mu_i\|^2 \epsilon_i^{-1}$$
$$= -(1+\theta_i)\|\mu_i\|^2 \epsilon_i^{-1} \tag{31}$$

By Assumption 3,

$$\|e_i\| = \|h_i + E_i K_i x_i\| \le (1+\theta_i)\rho_i(x_i, \beta_i, t) \tag{32}$$

The interconnected portion is bounded by, using Assumption 4,

$$x_i^T P_i \sum_{j=1}^{N} g_{ij}(x_j, t) \le (1+\theta_i)\lambda_M(P_i)\|x_i\| \sum_{j=1}^{N}\left(a_{ij}\|x_j\| + b_{ij}\right) \tag{33}$$

Combining Eqs. (29), (30), (32), and (33), as $\|\mu_i\| > \epsilon_i$,

$$\dot V_{i1} \le -\lambda_m(Q_i)\|x_i\|^2$$
$$- 2(1+\theta_i)\|B_i^T P_i x_i\|\left[\rho_i(x_i, \hat\beta_i, t) - \rho_i(x_i, \beta_i, t)\right]$$
$$+ 2(1+\theta_i)\lambda_M(P_i)\|x_i\| \sum_j \left(a_{ij}\|x_j\| + b_{ij}\right) \tag{34}$$

Similarly, as $\|\mu_i\| \le \epsilon_i$,

$$\dot V_{i1} \le -\lambda_m(Q_i)\|x_i\|^2 + \epsilon_i$$
$$- 2(1+\theta_i)\|B_i^T P_i x_i\|\left[\rho_i(x_i, \hat\beta_i, t) - \rho_i(x_i, \beta_i, t)\right]$$
$$+ 2(1+\theta_i)\lambda_M(P_i)\|x_i\| \sum_j \left(a_{ij}\|x_j\| + b_{ij}\right) \tag{35}$$

Now consider the time derivative of V_{i2} along the solution. Adopting Eq. (8), it follows that

$$\dot V_{i2} = 2(1+\theta_i)(\hat\beta_i - \beta_i)^T L_{i1}^{-1}\dot{\hat\beta}_i$$
$$= 2(1+\theta_i)(\hat\beta_i - \beta_i)^T \|B_i^T P_i x_i\| \frac{\partial}{\partial \beta_i}\rho_i^T(x_i, \hat\beta_i, t)$$
$$- 2(1+\theta_i)(\hat\beta_i - \beta_i)^T L_{i1}^{-1} L_{i2}\hat\beta_i$$
$$= 2(1+\theta_i)(\hat\beta_i - \beta_i)^T \|B_i^T P_i x_i\| \frac{\partial}{\partial \beta_i}\rho_i^T(x_i, \hat\beta_i, t)$$
$$- 2(1+\theta_i)(\hat\beta_i - \beta_i)^T L_{i1}^{-1} L_{i2}(\hat\beta_i - \beta_i)$$
$$- 2(1+\theta_i)(\hat\beta_i - \beta_i)^T L_{i1}^{-1} L_{i2}\beta_i \tag{36}$$

Combining Eqs. (34–36) yields

$$\dot V \le \sum_{i=1}^{N}\Bigg[-\gamma_i\lambda_m(Q_i)\|x_i\|^2 + \gamma_i\epsilon_i - 2\gamma_i(1+\theta_i)\|\alpha_i\|$$
$$\frac{\partial \rho_i^T}{\partial \beta_i}(x_i, \hat\beta_i, t)(\beta_i - \hat\beta_i) + 2\gamma_i(1+\theta_i)\|\alpha_i\|(\hat\beta_i - \beta_i)^T$$
$$\frac{\partial \rho_i}{\partial \beta_i}(x_i, \hat\beta_i, t) + 2\gamma_i(1+\theta_i)\lambda_M(P_i)\|x_i\|\sum_j\left(a_{ij}\|x_j\| + b_{ij}\right)$$
$$- 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|^2$$
$$+ 2\gamma_i\bar\beta_i(1+\theta_i)\lambda_M(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|\Bigg]$$
$$= \sum_{i=1}^{N}\Bigg[-\gamma_i\lambda_m(Q_i)\|x_i\|^2 + \gamma_i\epsilon_i + 2\gamma_i(1+\theta_i)\lambda_M(P_i)$$
$$\times \|x_i\|\sum_j\left(a_{ij}\|x_j\| + b_{ij}\right)$$
$$- 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|^2$$
$$+ 2\gamma_i\bar\beta_i(1+\theta_i)\lambda_M(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|\Bigg] \tag{37}$$

where $\bar\beta_i = \|\beta_i\|$. A further analysis shows

$$\sum_{i=1}^{N}\left[-\gamma_i\lambda_m(Q_i)\|x_i\|^2 + 2\gamma_i(1+\theta_i)\lambda_M(P_i)\|x_i\|\sum_j a_{ij}\|x_j\|\right]$$
$$= \sum_i -\gamma_i\lambda_m(Q_i)\|x_i\|^2 + \sum_{i,j} 2\gamma_i(1+\theta_i)\lambda_M(P_i)a_{ij}\|x_i\|\,\|x_j\|$$
$$= -\tfrac{1}{2}\kappa^T(\Omega T + T^T\Omega)\kappa$$
$$= -\kappa^T S \kappa \tag{38}$$

where $\kappa = [\|x_1\|\ \|x_2\| \ldots \|x_N\|]^T$ and, hence, $\|\kappa\|^2 = \Sigma_i\|x_i\|^2 = \|x\|^2$. This result enables us to show the following:

$$\dot V \le -\kappa^T S\kappa + \sum_{i=1}^{N}\Bigg[\gamma_i\epsilon_i + 2\gamma_i(1+\theta_i)\lambda_M(P_i)\|x_i\|\sum_j b_{ij}$$
$$- 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|^2$$
$$+ 2\gamma_i\bar\beta_i(1+\theta_i)\lambda_M(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|\Bigg]$$
$$\le -\lambda_m(S)\|\kappa\|^2 + \sum_i\Bigg[\gamma_i\epsilon_i + 2\gamma_i(1+\theta_i)\lambda_M(P_i)\|x_i\|\sum_j b_{ij}$$
$$- 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|^2$$
$$+ 2\gamma_i\bar\beta_i(1+\theta_i)\lambda_M(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|\Bigg] \tag{39}$$

Recalling that $\|\kappa\|^2 = \Sigma_i\|x_i\|^2$, we have

$$-\lambda_m(S)\|\kappa\|^2 = \sum_i -\lambda_m(S)\|x_i\|^2 \tag{40}$$

Moreover, recall that $\xi = [x^T\hat\beta^T - \beta^T]^T$ and, hence, $\|\xi\|^2 = \|x\|^2 + \|\hat\beta - \beta\|^2$, $\|x\| \le \|\xi\|$, and $\|\hat\beta - \beta\| \le \|\xi\|$,

$$\sum_i\left[-\lambda_m(S)\|x_i\|^2 - 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|^2\right]$$
$$\le -\sigma_1\|\xi\|^2 \tag{41}$$

and

$$\sum_i\Bigg[2\gamma_i(1+\theta_i)\lambda_M(P_i)\|x_i\|\sum_j b_{ij} + 2\gamma_i\bar\beta_i(1+\theta_i)$$
$$\lambda_M(L_{i1}^{-1}L_{i2})\|\hat\beta_i - \beta_i\|\Bigg] \le \sigma_2\|\xi\| \tag{42}$$

where

$$\sigma_1 = \left[\sum_i \sigma_{1i}^2\right]^{1/2}$$
$$\sigma_{1i} = \min\left\{\lambda_m(S),\ 2\gamma_i(1+\theta_i)\lambda_m(L_{i1}^{-1}L_{i2})\right\}$$
$$\sigma_2 = 2N^{1/2}\max_i\{\sigma_{2i}\}$$
$$\sigma_{2i} = \max\left\{2\gamma_i(1+\theta_i)\lambda_M(P_i)\sum_j b_{ij},\ 2(1+\theta_i)\bar\beta_i\gamma_i\lambda_M(L_{i1}^{-1}L_{i2})\right\}$$

Using Eqs. (37–42) for Eq. (37) yields

$$\dot V \le -\sigma_1\|\xi\|^2 + \sigma_2\|\xi\| + \sum_i \gamma_i\epsilon_i \tag{43}$$

almost everywhere on $[t_0, t_1)$. We conclude that

$$\dot V < 0 \tag{44}$$

for all ξ such that

$$\sigma_1\|\xi\|^2 - \sigma_2\|\xi\| - \sigma_3 > 0 \tag{45}$$

or

$$\| \xi \| > \frac{\sigma_2}{2\sigma_1} + \left[\left(\frac{\sigma_2}{2\sigma_1} \right)^2 + \frac{\sigma_3}{\sigma_1} \right]^{\frac{1}{2}} \triangleq \underline{d} \tag{46}$$

where $\sigma_3 = \Sigma_i \gamma_i \epsilon_i$. This implies that $\| \xi \|$ should be sufficiently large to assure $\dot{V} < 0$. Note again that both σ_1 and σ_2 are related to the possible bounds. That σ_{1i} being positive is assured if all of the principal minors of the test matrix T are positive. Properties 2–5 then follow.[22,23] ■

Remark 3: Notice the difference between properties 1–5 and Ref. 22, which requires that properties 4 and 5 hold for an arbitrary $\underline{d} > 0$. In the current case, the value of $\underline{d}$ is lower bounded by $\underline{d}_l$ where

$$\underline{d}_l = \lim_{\epsilon_i \to 0, \forall i} \underline{d} = \frac{\sigma_2}{\sigma_1}$$

Remark 4: The adaptive scheme (8) belongs to the so-called leakage type.[24] The second term on the right-hand side determines the amount of leakage. If the system performance is satisfactory (hence, the first term on the right-hand side may be small) and the estimated bound is large (hence, the second term is large), then it is possible that $\hat{\beta}_i$ is negative and the adaptive parameter $\hat{\beta}_i$ decreases. This self-adjustment mechanism prevents the estimated uncertainty, which is bound to be unnecessarily large. This prevents the control magnitude from becoming too large.

Remark 5: The decentralized adaptive robust control (9) can be viewed as an extension of the decentralized robust control developed earlier,[1,23] where the possible bound of the uncertainty must be determined a priori. In the present setup, the bound does not need to be determined and an on-line adaptive scheme is proposed. A practical advantage of this is that less control effort is required. If the bound is to be determined a priori, then one may sometimes be forced to choose a conservative estimation due to the lack of knowledge of the uncertainty. This then results in excessive control action. The present new design is mainly proposed to overcome this difficulty. We also recall that the first work on adaptive robust control was in Ref. 25, where a centralized control system was considered. However, the current adaptive scheme introduces the additional leakage term.

III. Uncertain Flexible Structure Model

The following strategy will be used to obtain the system model. First, the actual flexible structure is proposed. Second, the postulated flexible structure is proposed. Third, the postulated flexible structure will be reduced to state-space form via the modal analysis method. Fourth, the actual model will be reduced to state-space form. Finally the uncertainties will be extracted form the actual model.

The actual flexible structure, which must be satisfied at every point p of a given domain D, is given by[7]

$$L^*\left[u(p,t)\right] + M^*(p)\frac{\partial^2 u(p,t)}{\partial t^2} + d_d(p,t) + \sum_{i=1}^{q} d_{ci}(t)\delta(p - p_i) = f(p,t) \tag{47}$$

where $u(p,t)$ is the displacement of any point p at time t; $L^*[\cdot]$, which contains uncertian stiffness distribution information, is a positive definite spatial differential operator of order $2o$; $M^*(p)$ is the uncertain spatial distributed mass operator; $d_d(p,t)$ is the distributed disturbance; $d_{ci}(t)$ is the uncertain amplitude of the concentrated disturbance acting at $p = p_i$; and $f(p,t)$ is the distributed control function.

At every point p_i of the boundary S of domain D, the displacement $u(p,t)$ must satisfy the following conditions:

$$B_{ci}\left[u(p_i,t)\right] = 0, \qquad i = 12,\ldots o \tag{48}$$

where $B_{ci}[\cdot]$ is a linear partial differential operator with derivatives that are normal and tangential to the boundary and whose order ranges from zero to $2o - 1$.

We now propose a postulated flexible structure from our perception of the actual flexible structure. The postulated flexible structure, which must be satisfied at every point p in domain D, is given as

$$L\left[u(p,t)\right] + M(p)\frac{\partial^2 u(p,t)}{\partial t^2} = f(p,t) \tag{49}$$

where $L[\cdot]$ is the self-adjoint positive definite spatial partial differential operator of order $2o$ containing the known stiffness distribution information, and $M(p)$ is the self-adjoint positive definite spatial operator consisting of the known distributed mass function.

The eigenvalue problem associated with Eq. (49) is

$$L[\phi_i] = \lambda_i M\phi_i, \qquad i = 1,2,\ldots \tag{50}$$

where ϕ_i is the eigenfunction associated with the ith mode, and λ_i is the eigenvalue associated with the ith mode. Assuming that the eigenvalues are distinct, the solution of Eq. (50) will yield an orthogonal eigenfunction that can be normalized by

$$\int_D M\phi_i\phi_j dD = \delta_{ij} \tag{51}$$

$$\int_D \phi_j L\,[\phi_i]\, dD = \lambda_i\delta_{ij} \tag{52}$$

where δ_{ij} is the Kronecker delta. The expansion theorem can be employed with

$$u(p,t) = \sum_{i=1}^{\infty}\phi_i(p)\eta_i(t) \tag{53}$$

where $\eta_i(t)$ are the ith mode time-dependent generalized coordinates, known as the modal amplitude. The postulated flexible structure can now be transformed into an infinite set of uncoupled ordinary differential equations segregated by mode:

$$\ddot{\eta}_i(t) + \eta_i(t)\lambda_i = \Psi_i(t), \qquad i = 1,2,\ldots \tag{54}$$

where $\Psi_i(t)$ is the generalized control force associated with the ith mode, which is given by

$$\Psi_i(t) = \int_D \phi_i(p)f(p,t)dD \tag{55}$$

We now convert Eq. (55) into the state-space form:

$$\dot{x}_i(t) = A_i x_i(t) + B_i\Psi_i(t), \qquad i = 1,2,\ldots \tag{56}$$

by introducing the modal state vector $x_i(t) = \{\nu_i(t),\ \eta_i(t)\}^T$, where $\nu_i(t) = \dot{\eta}_i(t)$, the system matrix

$$A_i = \begin{bmatrix} 0 & -\lambda_i \\ 1 & 0 \end{bmatrix} \tag{57}$$

and the input matrix $B_i = \{1\ \ 0\}^T$.

To reduce the actual flexible structure to modal space form, we use the postulated system's eigenfunctions because they are the design perception of the system. Following the same procedure as for the postulated system yields

$$M^*\ddot{\eta}(t) + K^*\eta(t) + \psi(t) = \Psi(t) \tag{58}$$

where the elements of the infinite-dimensional square matrices M^* and K^* can be represented by

$$M^*_{ij} = \int_D M^*(p)\phi_i\phi_j dD \tag{59}$$

$$K_{ij}^* = \int_D \phi_i L^*[\phi_j]dD \tag{60}$$

Moreover, the infinite-dimensional modal displacement vector is $\eta(t) = \{\eta_1(t), \eta_2(t), \dots\}^T$, the infinite-dimensional modal vector is $\Psi(t) = \{\Psi_1(t), \Psi_2(t), \dots\}^T$, and the disturbance is consolidated into

$$\psi_i(t) = \int_D \phi_i(p) d_d(p,t) dD + \sum_{i=1}^{q} \phi_i(p_i) d_{ci}(t) \tag{61}$$

and the infinite-dimensional disturbance vector is $\psi(t) = \{\psi_1(t), \psi_2(t), \dots\}^T$. To show the difference between the postulated and actual flexible structure, consider the following matrix decomposition

$$M^{*-1} = I + \Delta M \tag{62}$$

$$K^* = \Lambda + \Delta K \tag{63}$$

where I is an infinite-dimensional identity matrix, $\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \dots]$, and ΔM and ΔK are real infinite-dimensional square matrices that implicitly relate the difference between the actual and postulated systems' mass and stiffness distributions, respectively. Substituting Eqs. (62) and (63) into Eq. (58) yields

$$\ddot{\eta} = -\Lambda\eta + \Delta C\eta + (I + \Delta M)\Psi - (I + \Delta M)\psi \tag{64}$$

where ΔC is an infinite-dimensional square matrix defined by

$$\Delta C = -\left[\Delta M\Lambda + \Delta K(I + \Delta M)\right] \tag{65}$$

System (64) can also be converted to the following form:

$$\begin{bmatrix} \dot{\nu}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{pmatrix} 0 & -\Lambda \\ I & 0 \end{pmatrix} \begin{bmatrix} \nu(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} \Delta C\eta(t) \\ 0 \end{bmatrix} + \left[\begin{pmatrix} I \\ 0 \end{pmatrix} + \begin{pmatrix} \Delta M \\ 0 \end{pmatrix}\right]\Psi(t) - \left[\begin{pmatrix} I \\ 0 \end{pmatrix} + \begin{pmatrix} \Delta M \\ 0 \end{pmatrix}\right]\psi(t) \tag{66}$$

We propose the following modal coordinate transformation:

$$\{\nu^T, \ \eta^T\}^T = T_I\{\nu_1, \eta_1, \nu_2, \eta_2, \dots\}^T = T_I\{x\}^T \tag{67}$$

where T_I is an infinite-dimensional square orthogonal linear transformation matrix and x is the infinite-dimensional state vector

$$x = \{\nu_1, \eta_1, \nu_2, \eta_2, \dots\}^T = \{x_1^T, x_2^T, \dots\}^T \tag{68}$$

The following modal state-space model of the actual flexible structure can be obtained:

$$\dot{x}(t) = Ax(t) + \Delta f\left(x(t),t\right) + \left\{B + \Delta B\left(x(t),t\right)\right\}\Psi(t) + G\left(x(t),t\right) \tag{69}$$

where A is the infinite-dimensional state coefficient matrix:

$$A = T_I^T \begin{pmatrix} 0 & -\Lambda \\ I & 0 \end{pmatrix} T_I = \mathrm{diag}\left[\begin{pmatrix} 0 & -\lambda_1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -\lambda_2 \\ 1 & 0 \end{pmatrix}\cdots\right] \tag{70}$$

$\Delta f(x(t),t)$ is the infinite-dimensional uncoupled modal uncertainty vector:

$$\Delta f = T_I^T \begin{bmatrix} \mathrm{diag}(\Delta C)\eta - \mathrm{diag}(I + \Delta M)\psi \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta C_{11}\eta_1 - (1 + \Delta M_{11})\psi_1 \\ 0 \\ \Delta C_{22}\eta_2 - (1 + \Delta M_{22})\psi_2 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{71}$$

B is the infinite-dimensional control coefficient uncertainty matrix:

$$B = \mathrm{diag}\left(\{1, 0\}^T, \ \{1, 0\}^T, \dots\right) \tag{72}$$

ΔB is the infinite-dimensional uncertain control matrix:

$$\Delta B = \mathrm{diag}\left(\{\Delta M_{11}, 0\}^T, \ \{\Delta M_{22}, 0\}^T, \dots\right) \tag{73}$$

and $G[x(t),t]$ is the infinite-dimensional tied (coupled) modal uncertainty vector:

$$G = T_I^T\left[\Delta C\eta + \Delta M(\Psi - \psi)\right] \tag{74}$$

We also note that $\Delta C_{ii} = 0$ and $\Delta M_{ii} = 0$. G can also be represented by

$$G\left(x(t),t\right) = \{G_1, 0, G_2, 0, \dots\}^T \tag{75}$$

where

$$G_i = \sum_{j=1, j\neq i}^{\infty} \Delta C_{ij}\eta_j + \Delta M_{ij}(\Psi_j - \psi_j) \tag{76}$$

The reason for converting into the state-space form (69) is for control purpose. It is clear, upon viewing the structures of the uncertain portions, as depicted in Eqs. (71), (73) and (76), that Assumptions 1 and 2 are both met. Equation (69) can also be viewed as a class of interconnected subsystems segregated and identifiable by modes. Each mode subsystem is represented by

$$\dot{x}_i(t) = A_i x_i(t) + \Delta f_i\left(x_i(t),t\right) + (B_i + \Delta B_i)\Psi_i(t) + \sum_{j=1, j\neq i}^{\infty} g_{ij}\left(x_j(t),t\right) \tag{77}$$

$i = 1, 2, \dots$, where the uncoupled uncertainty matrix of dimension 2×1 is

$$\Delta f_i\left(x_i(t),t\right) = \left\{\left[\Delta C_{ii}\eta_i - (1 + \Delta M_{ii})\psi_i\right], 0\right\}^T \tag{78}$$

the uncertain control matrix of dimension 2×1 is

$$\Delta B_i = \{\Delta M_{ii}, 0\}^T \tag{79}$$

and the jth mode coupled (i.e., jth coupled to ith mode) uncertainty matrix of dimension 2×1 is

$$g_{ij}\left(x_j(t),t\right) = \left\{\left[\Delta C_{ij}\eta_j + \Delta M_{ij}(\Psi_j - \psi_j)\right], 0\right\}^T \tag{80}$$

Note that the difference between the ith mode state equations (56) and (77) is clearly defined in Eqs. (78–80); hence, when g_{ij}, ΔB_i, and Δf_i are zero, Eq. (77) reduces to Eq. (56).

IV. Flexible Structure with Decentralized Adaptive Robust Control

Because of modal analysis, the ith modal state can be extracted from sensor data by modal filters:

$$\eta_i(t) = \int_D M(p)\phi_i(p)u(p,t)dD \tag{81}$$

$$\nu_i(t) = \int_D M(p)\phi_i(p)\dot{u}(p,t)dD \tag{82}$$

Notice that the modal filters are based on the postulated system's orthonormal eigenfunctions. The actual control force is also synthesized from the postulated system's orthonormal eigenfunctions by

$$f(p,t) = \sum_{i=1}^{\infty} M(p)\phi_i(p)\Psi_i(t)dD \tag{83}$$

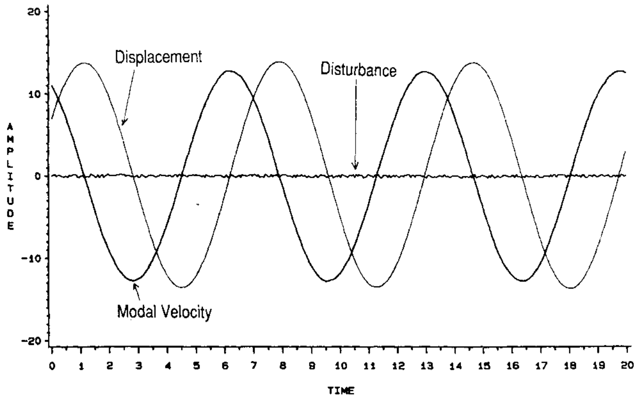

Fig. 1 Mode 2, without control.

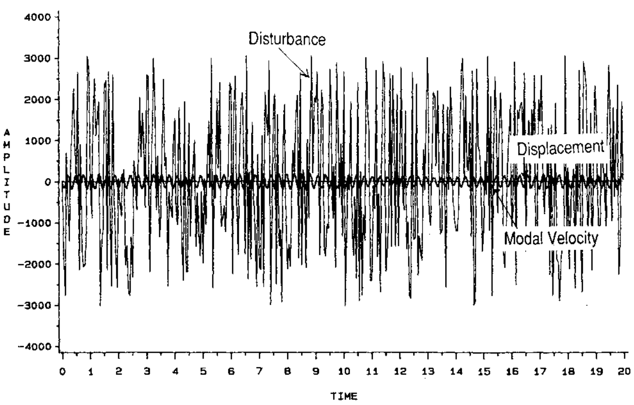

Fig. 2 Mode 7, without control.

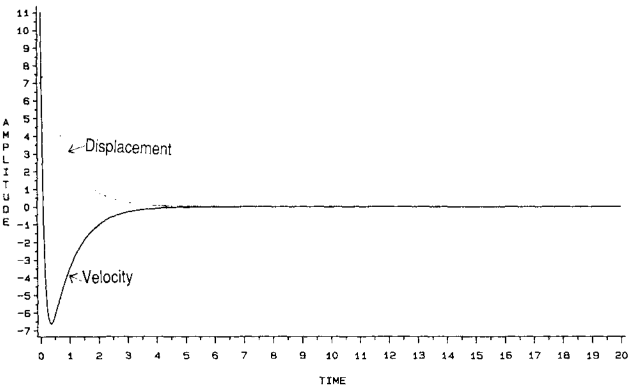

Fig. 3 Mode 2, with adaptive robust control.

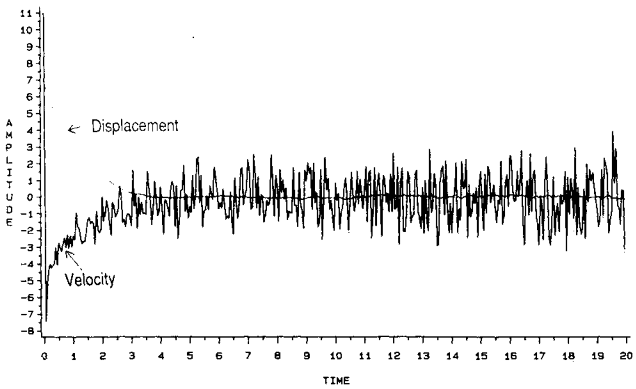

Fig. 4 Mode 7, with adaptive robust control.

Table 1 Gain matrix elements

Mode i	K_{i1}	K_{i2}
1	-7	-12.23
2	-7	-11.44
3	-7	-6.58
4	-7	-1.00
5	-7	-1.00
6	-7	-1.00
7	-7	-1.00

Table 2 Adaptive scheme design parameters

Mode i	L_{i1}	L_{i2}
1	$2.0I_2$	$0.1I_2$
2	$2.0I_2$	$0.1I_2$
3	$7.0I_2$	$0.49I_2$
4	$7.0I_2$	$0.7I_2$
5	$7.0I_2$	$0.49I_2$
6	$7.0I_2$	$0.7I_2$
7	$7.0I_2$	$0.7I_2$

Next, consider the problems involved when a finite number of system's modes are controlled. In practice, only a limited number of modes can be retained for control (9). Hence, the modal state model is truncated to include only a finite number of lower modes, a process referred to as discretization. Participation of the higher modes in the overall system response $u(p,t)$ tends to be less significant due to their decreased amplitudes[8]; therefore, the number of modes for control implementation is determined by performance desire. A discussion on the general issues related to discrete mode control implementation and control spillover can be found in Refs. 7-9 and 26. Simulation study shows that the current control scheme is robust to spillover effects. Control (9) and the decentralized leakage-type adaptive algorithm will be used to subdue the response of a tapered cantilever beam that is governed by

$$\frac{\partial^2}{\partial p^2}\left[EI^*(p)\frac{\partial^2 u(p,t)}{\partial p^2}\right] + M^*(p)\frac{\partial^2 u(p,t)}{\partial t^2} = 0 \tag{84}$$

with the following properties:

Mass distribution:

$$M^*(p) = 1.1 - 0.05p \tag{85}$$

Stiffness distribution:

$$EI^*(p) = 1.2 - 0.07p \tag{86}$$

and a time-varying random disturbance between -1 and 1.

The modal state was truncated to include the first seven vibratory modes. All seven of the vibratory modes were controlled. A tapered cantilever beam was chosen as a postulate flexible structure whose parameters are

Mass distribution:

$$M(p) = 1 - 0.04p \tag{87}$$

Stiffness distribution:

$$EI(p) = 1 - 0.04p \tag{88}$$

The Rayleigh-Ritz method was used to approximate the postulated system's eigensolution from the closed-form eigensolution for a uniform cantilever beam.[27] The adaptive robust control was designed on the basis of the postulated flexible beam. The postulated eigenvalues were used to obtain the feedback gains given in Tables 1 and 2. A 2×2 identity matrix was chosen as Q_i for the Lyapunov equation, and then the values of P_i were computed.

In the simulations, three parameters are manipulated to alter the system performance. These are L_{i1}, L_{i2}, and ϵ_i. Different combinations of the parameters result in a tradeoff between overshoot, settling time, and control effort. The results by using one set of parameters for the seven modes of the preceding flexible beam are presented here. The value of ϵ_i was chosen to be 0.001 for all seven modes. The constant gain matrix $K_i = [K_{i1}\ K_{i2}]$ and the adaptive scheme design parameters L_{i1} and L_{i2} used for this set are given in Tables 1 and 2. Note that, in this flexible beam, it can be shown that $\rho_i = \beta_{i1} + \beta_{i2}\|x_i\|$ and, hence, $s_i = 2$, $i = 1,2,\ldots,7$. Assumptions 3 and 4 are met. Figures 1 and 2 depict the performance of modes 2 and 7 without using any control. In each figure, modal displacement, modal velocity, and external disturbance are plotted vs time. Figures 3 and 4 are the corresponding mode performance as the adaptive robust control is applied. In each figure, modal displacement and velocity are plotted vs time. Significant improvement of performance is observed.

V. Conclusions

From an application point of view, the decentralized adaptive robust control possesses two distinct advantages. First, one does not need to study in detail all of the physical properties of the uncertainties and the system operation conditions in order to obtain the bound of uncertainty. The on-line adaptive scheme is able to track the performance and then determines a suitable estimation. Second, the estimation of the bound does not converge to the true bound. This is in fact plausible since otherwise high control magnitude may be inevitable. The leakage-type adaptive scheme is able to decrease the bound estimation once the system performance is considered satisfactory. This ensures a reasonable control strategy. The two advantages enable the decentralized adaptive robust control to be applicable to the uncertain flexible beam. If one intends to estimate the bound a priori (hence, use Ref. 1) instead of using the adaptive scheme for the example used here, our calculation shows the bound is about 20 times as large as the present case. The resulting control magnitude is beyond any reasonable threshold.

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